CS4400 Assignment 6 Supplement: Control Stacks for Simply-Typed λ-Calculus

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Expressions and values:

$$\begin{array}{rcl} e & ::= & x \mid \lambda x.e \mid e_1\ e_2 \\ v & ::= & \lambda x.e \end{array}$$

S terminal: state S is *terminal* (cannot transition).
Stack syntax:

$$\begin{array}{rcl} k & ::= & \epsilon \mid k; f \\ f & ::= & (\square\ e) \mid ((\lambda x.e)\ \square) \end{array}$$

States:

$$S \quad ::= \quad k \rhd e \mid k \lhd v$$

Terminal states have an empty stack and a value:

$$\frac{}{\epsilon \lhd v\ \text{terminal}}$$

State transition rules:
$\boxed{S \mapsto S'}$

$$\frac{}{k \rhd \lambda x.e \mapsto k \lhd \lambda x.e}$$

$$\frac{}{k \rhd (e_1\ e_2) \mapsto k; (\square\ e_2) \rhd e_1}$$

$$\frac{}{k; (\square\ e_2) \lhd (\lambda x.e) \mapsto k; ((\lambda x.e)\ \square) \rhd e_2}$$

$$\frac{}{k; ((\lambda x.e)\ \square) \lhd v \mapsto k \rhd [v/x]e}$$